Application of a preconditoned truncated Newton method to full waveform inversion

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Abstract

Full Waveform Inversion (FWI) is a powerful seismic imaging method, based on the iterative minimization of the distance between simulated and recorded wavefields. The inverse Hessian operator related to this misfit function plays an important role in the reconstruction scheme. As conventional methods use direct approximations of this operator, we investigate an alternative optimization scheme: the truncated Newton method. This two-nested-loops algorithm is based on the resolution of the Newton linear system through a matrix-free iterative solver at each outer iteration. On the 2D BP 2004 model widely used as a benchmark for FWI, the contrasts in wave velocities between salt structures and the upper water layer generate high amplitude multiple reflections. These multiple reflections strengthen the need for quite accurate approximation of the inverse Hessian operator and the truncated Newton method is shown to outperform than more conventional algorithms (l-BFGS, nonlinear conjugate gradient).

Introduction

Full Waveform Inversion is a seismic imaging method dedicated to the computation of high resolution quantitative estimates of subsurface parameters such as pressure wave velocity, shear wave velocity, attenuation, or density. This method consists in computing a subsurface model p which minimizes a misfit function $f(p)$, defined by

$$f(p) = \frac{1}{2}\sum_{s=1}^{S} \|u_s(p) - d_s\|^2, \tag{1}$$

which measures the distance between the simulated wavefields $u_s(p)$ and the actual recorded wavefields d_s. Despite its early introduction in the 80s, only the recent development of computational capacities (computer clusters) and acquisition systems (wide-azimuth wide-offset broadband seismic surveys) have made possible its application to real data in oil and gas industry.

In this study, we particularly focus on the minimization method which is used to solve the FWI problem. As the large number of discrete unknowns prevents from using global optimization methods, state-of-the-art methods are local gradient-based methods such as the nonlinear conjugate gradient (CG) or the l-BFGS method. From an initial subsurface model p_0, a sequence p_k is built such that

$$p_{k+1} = p_k + \alpha_k \Delta p_k, \tag{2}$$

where α_k is computed through a linesearch method and Δp_k is the descent direction

$$\Delta p_k = -Q_k \nabla f(p_k). \tag{3}$$

The matrix Q_k is an approximation of the inverse Hessian matrix $(\nabla^2 f(p_k))^{-1}$. Pratt [2] clearly demonstrates the crucial role played by this operator in the FWI reconstruction scheme:

- it acts as a deconvolution operator that accounts for the limited bandwidth of the seismic data and corrects for the loss of amplitude of poorly illuminated subsurface parameters;
- it helps to remove artifacts that the second order reflected waves may generate on the gradient descent direction.

For multi-parameters FWI, the off-diagonal blocks of the Hessian matrix should also account for the trade-off between different classes of parameters. This suggests that it should be crucial to account accurately for the inverse Hessian operator within the minimization schemes, and leads us to the investigation of the truncated Newton method for FWI.

Methodology

The truncated Newton method only differs from standard descent method by the strategy used to compute the descent direction. Instead of using an approximation of the inverse Hessian operator, the descent direction Δp_k is computed through the resolution of the Newton linear system

$$\nabla^2 f(p_k)\Delta p_k = -\nabla f(p_k), \tag{4}$$

using a matrix-free CG solver, which results in a two-nested loops algorithm (inner linear CG iterations for the computation of Δp_k through (4) and outer non-linear iterations for the construction of the sequence p_k through (2)). The incomplete resolution of the linear system (4) is referred as the truncation strategy. This presents several advantages over conventional procedures:

- the inverse Hessian operator is more accurately accounted for;
- the approximations of the inverse Hessian operator developed for the standard methods can be reintroduced within this framework as preconditioners of the linear system (4);
- the method is well suited for applications where the misfit function change over the iterations, as for instance using random combinations of data-sets d_s (source encoding techniques);
- the truncation strategy can be seen as an intrinsic regularization of the FWI problem (of particular interest for the interpretation of noisy data).

An efficient implementation of this algorithm for FWI is fully described in [1]. It mainly relies on the reduction of the computation cost associated with the inner loop. This is achieved using:

- second-order adjoint-state formulae for the computation of Hessian-vector products;
- an adaptive stopping criterion for the inner iterations, related to the truncation strategy, a crucial issue;
- an efficient preconditioning method based on the approximation of the diagonal terms of the inverse Hessian operator.

Numerical results

We compare the truncated Newton method with the nonlinear CG method and the l-BFGS method using the same preconditioning technique. This comparison is performed on the BP 2004 model, which exhibits complex subsurface patterns related to the presence of salt structures (figure 1). The high contrast in wave velocity between the water layer and these salt structures are responsible for the presence of high amplitude multi-reflected waves which renders an accurate estimation of the inverse Hessian operator crucial for a stable reconstruction of the subsurface model. These experiments are performed under the acoustic approximation, and we aim at recovering the pressure wave velocity model. We solve the wave equation in the frequency-domain (the forward problem is then described by the Helmholtz equation) and we adopt the so-called hierarchical approach: 6 groups of overlapping frequencies are inverted from 2.5 Hz to 20.5 Hz. The initial model p_0 (figure 1) is a smooth version of the exact one which shall be obtained using conventional tomography methods. The results provided by the three optimization schemes are presented in figure 2. As it can be seen, only the truncated Newton method provides a reliable estimation in this specific case of high contrasts.

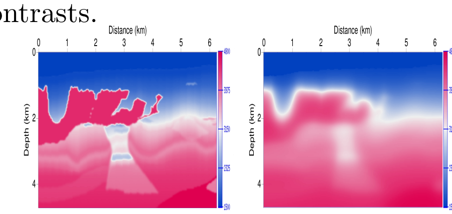

Figure 1. BP 2004 model (left), initial model (right).

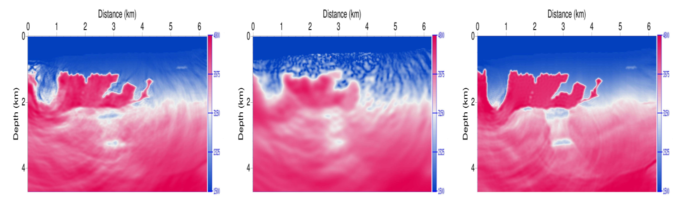

Figure 2. Nonlinear CG result (left), l-BFGS result (center), truncated Newton result (right).

Conclusion and perspectives

An accurate estimation of the inverse Hessian operator within the FWI reconstruction scheme is of particular importance for computing accurate estimations of the subsurface parameters. In the 2D acoustic approximation, when high amplitude multiple reflected waves have to be interpreted, the truncated Newton method provides a better alternative to conventional optimization methods. Application to real data is now the next step for investigating the interest of this method for FWI. This method will be also further investigated in anisotropic and elastic contexts, for multi-parameter reconstructions, in 2D and 3D experiments. The coupling of this method with source encoding strategies shall also be investigated.

References

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